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Calculated Uncertainty of Temperature Due to the Size-of-Source Effect in Commercial Radiation Thermometers

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Abstract The article evaluates the uncertainty in the temperature indicated by a radiation thermometer with a direct readout in temperature, due to the uncertainty in measuring the size-of-source effect (SSE) by the so-called "direct method." Radiation thermometers of this type are the ones most frequently used in practice. The uncertainty of the SSE characteristic is usually not a useful quantity to report to users of commercial radiation thermometers. Instead, they would prefer to know the uncertainty in the measured temperature that results from the uncertainty of the SSE characteristic, and this will be the result of our analysis. The user of a direct reading radiation thermometer will be able to take into account the uncertainty of temperature due to the SSE, if a target with known dimensions is measured. The uncertainty in temperature due to the SSE of analyses based on Planck's law and its approximation, Wien's law is compared.

Keywords Background radiation · Size-of-source effect · Uncertainty of temperature

1 Introduction

In the calibration of radiation thermometers, one of the contributors to the total uncertainty of measurement is the dependence on target size, the phenomenon known as the size-of-source effect (SSE), which results from radiation being scattered into, or out

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of, the thermometer's nominal target area. Many radiation thermometers, especially commercial thermometers operating in the infrared spectrum and used to measure temperatures below 600°C, suffer from a poor SSE characteristic. To determine the SSE, two methods, known as the direct and indirect methods, are available. In the direct method, a radiation thermometer is focused on a blackbody (radiation source) of variable-diameter aperture. The ratio between the signal at a given radius and the signal at the maximum radius is a measure of the SSE. To measure the characteristic of the SSE by the direct method, we developed a system that is based on a water-cooled holder of aluminum plates with apertures of different diameters. We measured the SSE characteristic of a direct-reading radiation thermometer by the direct method at different blackbody temperatures. Corrections for the measured SSE were evaluated in terms of the measurement uncertainty, expressed in terms of temperature.

2 Direct Method for Determining the SSE

In the direct method, a radiation thermometer is focused on a circular aperture placed in front of a stable radiation source, usually a blackbody. Measurements are made with different diameter apertures. The SSE at radius r is the ratio $\sigma_S(r)$ between the signal $S(r, L)$ at radius $r$and the signal $S(\infty, L)$ at infinite radius:

$$\sigma_S(r) = \frac{S(r, L)}{S(\infty, L)}. \tag{1}$$

In practice, we cannot realize an infinite radius; therefore, we measure the SSE as a function of radius in a limited range from $r_{min} \leq r \leq r_{max}$, where r_{max} usually represents the radius of the blackbody aperture. Thus, we determine the SSE as the ratio of signals at radii r and r_{max}:

$$\sigma_S(r, r_{max}) = \frac{\sigma_S(r)}{\sigma_S(r_{max})} = \frac{S(r, L)}{S(r_{max}, L)}. \tag{2}$$

So far, we have considered the background radiation arising from the aperture and its surroundings to be negligible compared to the radiation from the source. When this is not the case, we have to take the background radiation into consideration. Besides the radiation from the source of temperature T and radius r, the detector signal depends also on the background radiation with temperature T_a. Taking into consideration the background radiation L_a, we can write the relation between the SSE and the measured signal as

$$S(r, L, L_a) = \sigma_S(r)S(\infty, L) + (1 - \sigma_S(r))S(\infty, L_a). \tag{3}$$

In practice, the detector signal at infinite radius $S(\infty, L)$ is replaced by the signal at the maximum radius $S(r_{max}, L)$. The background radiation $S(\infty, L_a)$ is measured by directing a radiation thermometer to several targets in the background, and the average signal from those targets is calculated. The SSE, corrected for background radiation, is written as

$$\sigma_{S,a}(r, r_{max}) = \frac{S(r, L, L_a) - S(\infty, L_a)}{S(r_{max}, L, L_a) - S(\infty, L_a)}. \tag{4}$$

Eq. 1–4 are summarized from [1]. We can neglect the background radiation when we measure temperatures higher than 200°C [2]. Then, Eq. 4 reduces to the basic equation for the SSE, Eq. 2.

2.1 System for Measuring the SSE

To measure the SSE, we need radiation sources with different diameters. It would be unusual and impractical for a laboratory to have many blackbodies for the same temperature range with different diameter apertures. Therefore, we developed a system to measure the SSE by placing 14 aperture plates of different diameters in front of a blackbody. In this way, we limit the blackbody radiation to a known aperture diameter. Unfortunately, the aperture plates were heated by convection and radiation from the blackbody, with the plates having smaller apertures being heated more than those of larger diameter. To ensure the temperature stability of all aperture plates, we fixed them to a holder that was cooled by liquid from a temperature-regulated bath. For successful cooling, and to ensure thermal uniformity, the aperture plates and their holder should be made from a material of high thermal conductivity. We selected aluminum with a thermal conductivity of $237\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$ for the aperture plates, while the holder was made of copper with a thermal conductivity of $401\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$. To reduce reflected radiation from the background, the aperture plates and the holder were painted with a special paint that had a very high emissivity of 0.91 in the wavelength range from 8 to 16 µm. The aperture plates were placed into a groove in the holder and tightened with screws; ensuring good thermal contact between the aperture plates and its holder. We need to consider the background radiation that originates mainly from the aperture plate when measuring the SSE at low temperatures; therefore, we measured the relative temperature rise of the aperture plate with the help of an infrared ear thermometer placed 1 mm from the aperture plate at nine different positions. The measurement system and results are presented in more detail in [3,4].

2.2 Measurement of the SSE Characteristics

We set the temperature of the blackbody while cooling the aperture plate holder with a bath set to 23°C. We do not need to measure the temperature of the blackbody accurately because the SSE is based on relative differences expressed as the ratio of the radiation at two different target sizes, while the blackbody is maintained at a fixed temperature. We measured the temperature and observed the temperature stability of the blackbody with a platinum resistance thermometer. We placed the radiation thermometer on a special stand, with the thermometer's optical axis parallel to the cavity axis of the blackbody. We focused a radiation thermometer at the minimum distance to assure reproducible measurements, in our case 50 cm. We started the measurement with the largest target (aperture plate with the largest aperture diameter) and progressed in sequence to the smallest target size. After every change, we waited

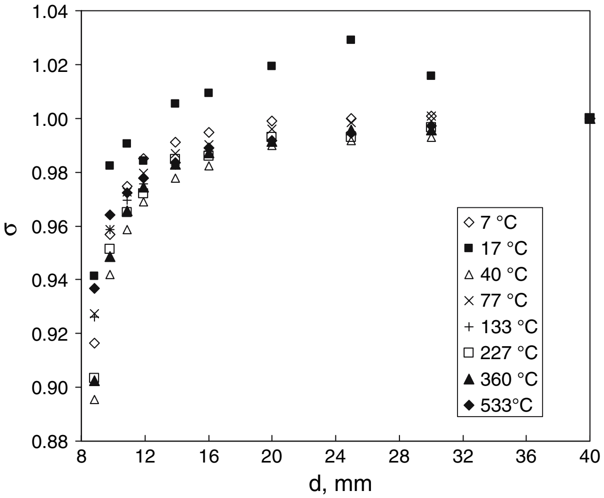

Fig. 1 Characteristics of the SSE corrected for background radiation for the Minolta Cyclops 300AF thermometer with a nominal target size of 9 mm, for targets with diameters from $d = 8.8$ to 40.0 mm

1 min for the temperature to stabilize. Then, we performed measurements with a radiation thermometer that was connected via a serial bus to a personal computer. The data acquisition software was developed using LabVIEW. The software stored important data, such as the date and time of measurement, the blackbody temperature, the target diameter, and the average measured temperature and its standard deviation.

For each target, we performed 30 measurements to assure stable conditions, especially with the smaller targets where the background radiation was relatively large. In calculating the SSE, we took the background radiation into consideration such that in Eq. 4 we applied the relation $S = aL$, where a is a constant of each radiation thermometer that characterizes the geometric capability of its optical system to transmit radiation, the absorptivity of the optical system, and the responsivity of the detector:

$$\sigma_{L,a}(d, d_{\max}) = \frac{L(d, T) - L(T_{\mathrm{a}})}{L(d_{\max}, T) - L(T_{\mathrm{a}})}, \tag{5}$$

where $L(T_{\mathrm{a}})$ represents the background radiation. The results of the calculation are presented in Fig. 1. The characteristic at 17°C deviated strongly from that at other temperatures as a consequence of subtracting two similar values in the numerator of Eq. 4 when the measured temperature was similar to the background temperature. Thus, the error of that particular SSE characteristic was larger. Similar considerations apply to the SSE characteristics at 7, 40, and 77°C, but to a lesser extent. The characteristics at 227 and 360°C in the range of target diameters from 8.8 to 40.0 mm agreed within 0.3%.

If we disregard the SSE characteristic at 17°C, the average of the other characteristics indicates that 92% of the incident radiation falls within the nominal target size of the radiation thermometer. The remaining 8% originates from diameters larger than 9 mm. The value for the nominal target size (9-mm diameter) was obtained by interpolating the results with targets of 8.8- and 9.8-mm diameter. The manufacturer provides neither the SSE specification nor the center wavelength. The nominal temperature range of the thermometer is −50 to 1,000°C.

3 Uncertainty of Temperature due to Uncertainty of the SSE

The uncertainty of a quantity that depends on many mutually independent parameters $y = f(x_1, x_2, \ldots, x_n)$, is given by the equation:

$$u^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i), \tag{6}$$

in which the quantity in brackets is the partial derivation of function f with respect to parameter x_i and $u(x_i)$ is the standard uncertainty of this parameter. In this way, we determine the uncertainty of the radiation $u(L)$ and the uncertainty of the SSE $u(\sigma)$. The uncertainty of the radiation, calculated from the measured temperature T by the method of equivalent radiation and assuming that the radiation thermometer is monochromatic [5], is given by the equation,

$$u(L) = \left|\frac{\partial L}{\partial T}\right| u(T). \tag{7}$$

If we consider Wien's law,

$$L_{\lambda,b}(\lambda, T) = \frac{c_{1\mathrm{L}}}{n^2\lambda^5}\left[\mathrm{e}^{\frac{c_2}{n\lambda T}}\right]^{-1}, \tag{8}$$

we can write Eq. 7 as

$$u(L) = L\left(\frac{c_2}{n\lambda T^2}\right) u(T). \tag{9}$$

In Eq. 8, $c_{1\mathrm{L}} = 2hc_0^2 = 1.191062 \times 10^{-16}\,\mathrm{W \cdot m^2 \cdot sr^{-1}}$ and $c_2 = hc_0/k = 0.014388\,\mathrm{m \cdot K}$ are the first and the second radiation constants, λ is the detection wavelength of the radiation thermometer, and n is the refraction coefficient of the medium through which the radiation is traveling. The radiation constants depend on the following constants: c_0 is the speed of light in vacuum, $h = 6.626176 \times 10^{-34}\,\mathrm{J \cdot s}$ is Planck's constant, and $k = 1.380662 \times 10^{-23}\,\mathrm{J \cdot K^{-1}}$ is Boltzmann's constant.

If we consider Planck's law,

$$L_{\lambda,b}(\lambda, T) = \frac{c_{1\mathrm{L}}}{n^2\lambda^5}\left[\mathrm{e}^{\frac{c_2}{n\lambda T}} - 1\right]^{-1} \tag{10}$$

we can write Eq. 7 as

$$u(L) = L^2 \mathrm{e}^{\frac{c_2}{n\lambda T}} \frac{c_2}{c_{1\mathrm{L}}} \frac{\lambda^4}{T^2} u(T). \tag{11}$$

The SSE depends on two variables, $L(d, T)$ in $L(d_{\max}, T)$, that are not completely independent, though we will assume that they are. By doing so, we are not underestimating the uncertainty and we can perform partial derivation by both variables indicated as L_1 and L_2:

$$u^2(\sigma) = \left(\frac{\partial\sigma}{\partial L_1}\right)^2 u^2(L_1) + \left(\frac{\partial\sigma}{\partial L_2}\right)^2 u^2(L_2). \tag{12}$$

Thus, we obtain

$$u^2(\sigma) = \left(\frac{1}{L_2}\right)^2 u^2(L_1) + \left(-\frac{L_1}{L_2^2}\right)^2 u^2(L_2). \tag{13}$$

By recognizing that $(L_1/L_2)^2$ is σ^2, we can rewrite Eq. 13 to obtain the relative uncertainty of the SSE, which is equal to the sum of the relative uncertainties of both radiations:

$$\frac{u^2(\sigma)}{\sigma^2} = \left(\frac{u(L_1)}{L_1}\right)^2 + \left(\frac{u(L_2)}{L_2}\right)^2. \tag{14}$$

We calculate the radiation from the measured temperature by the method of equivalent radiation [2]; therefore, we consider Eq. 9 and obtain the uncertainty of the SSE at a certain target diameter:

$$\frac{u(\sigma)}{\sigma} = \sqrt{\left(\frac{c_2}{n\lambda T_1^2}\right)^2 u^2(T_1) + \left(\frac{c_2}{n\lambda T_2^2}\right)^2 u^2(T_2)}. \tag{15}$$

The uncertainty in Eq. 15 depends on λ, c_2, T_1 measured at diameter d, T_2 measured at maximum diameter $d_{\max}$, and uncertainties of both temperatures, $u(T_1)$ in $u(T_2)$, that are usually the same because they are measured with the same thermometer. The absolute values of T_1 and T_2 are very similar. If we consider that they are the same, we can simplify Eq. 15 to

$$\frac{u(\sigma)}{\sigma} = \sqrt{2}\frac{c_2}{n\lambda T^2}u(T), \tag{16}$$

where for the uncertainty of temperature we take into account the uncertainty at the smaller target diameters, because it is larger and contributes more to the uncertainty.

If we consider Eq. 11 in Eq. 14 and assume that $T_1 \cong T_2$, we get for the relative uncertainty of the SSE:

$$\frac{u(\sigma)}{\sigma} = \sqrt{2}\frac{c_2}{n\lambda T^2}\frac{\mathrm{e}^{\frac{c_2}{n\lambda T}}}{\mathrm{e}^{\frac{c_2}{n\lambda T}} - 1}u(T). \tag{17}$$

Table 1 Differences between uncertainties of temperature of a radiation thermometer with the direct reading of temperature due to uncertainty of the size-of-source effect calculated with respect to Wien's or Planck's law

ϑ (°C)	u_{Wien} $(\sigma)/\sigma$	u_{Planck} $(\sigma)/\sigma$	u_{Wien} (°C)	u_{Planck} (°C)	$u_{\text{Wien}} - u_{\text{Planck}}$ (°C)	$\{u_{\text{Wien}} - u_{\text{Planck}}(°\text{C})\}/$ T(K)
7	0.00223	0.00225	0.0947	0.0947	0.00000	-1.0×10^{-8}
17	0.00163	0.00165	0.0741	0.0741	−0.00001	-1.8×10^{-8}
40	0.00139	0.00141	0.0737	0.0737	0.00001	2.3×10^{-8}
77	0.00086	0.00088	0.0570	0.0570	0.00000	-2.4×10^{-9}
133	0.00056	0.00059	0.0502	0.0502	0.00000	5.7×10^{-9}
227	0.00306	0.00330	0.4134	0.4133	0.00006	1.2×10^{-7}
360	0.00138	0.00158	0.3000	0.2982	0.00173	2.7×10^{-6}
533	0.00085	0.00105	0.2974	0.2956	0.00178	2.2×10^{-6}

For the typical user of a radiation thermometer, the uncertainty of temperature due to the uncertainty of SSE is the important quantity. From Eq. 16,

$$u(T) = \frac{1}{\sqrt{2}} \frac{n\lambda T^2}{c_2} \frac{u(\sigma)}{\sigma}. \tag{18}$$

Similarly, from Eq. 17,

$$u(T) = \frac{1}{\sqrt{2}} \frac{n\lambda T^2}{c_2} \frac{\mathrm{e}^{\frac{c_2}{n\lambda T}} - 1}{\mathrm{e}^{\frac{c_2}{n\lambda T}}} \frac{u(\sigma)}{\sigma}. \tag{19}$$

The differences in the uncertainty of temperature, based on results of the Minolta/Land Cyclops 300 AF commercial radiation thermometer, considering Wien's law in Eq. 18 and Planck's law in Eq. 19, are presented in Table 1. The uncertainties increase at 227°C because the resolution of the thermometer is 1°C above 200°C and 0.1°C below 200°C. It is obvious that the differences are negligible. This means that the user of a radiation thermometer that reads temperature directly can determine the uncertainty of temperature due to the uncertainty of the SSE by considering Wien's law rather than Planck's law.

4 Conclusion

The SSE is usually measured for targets larger than the nominal target size of a radiation thermometer. If we do not know the SSE, we should assume a target size of at least twice the nominal target size stated by the manufacturer. If we know the SSE characteristic corrected for background radiation, we are able to calculate the fraction of radiance that originates outside the nominal target size and determine its temperature-equivalent influence. The results show that the SSE is smaller at higher temperatures than at lower temperatures; therefore, we should determine the SSE at the highest temperature of the thermometer's measuring range. In our case, for a nominal target size of 9-mm diameter, the SSE at 533°C amounts to approximately 0.2%, while at 7°C it amounts to 50%. Based on this, we can conclude that we should perform measurements at higher temperatures within the measuring range of the radiation thermometer to ensure the correct determination of the SSE.

Characterization of the SSE enables more accurate measurements, if we use the information to correct measured temperatures for the influence of the SSE. Every correction has an associated uncertainty. In calculating the uncertainty due to the SSE, the approximations presented in this article should be carefully applied. Unfortunately, for users of radiation thermometers with a direct reading of temperature, the uncertainty of the SSE is not easily applied; instead, the uncertainty of temperature due to the uncertainty of the SSE is the useful quantity of interest. Therefore, a calibration laboratory should calculate and provide the user with the uncertainty in temperature resulting from the uncertainty of the SSE. This uncertainty can be calculated based on Wien's or Planck's law. The presented results showed that the differences from choosing Wien's or Planck's law when calculating the uncertainty in temperature due to the uncertainty of the SSE were negligible.

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